Resit 'Take-home exam' *Introduction to Dynamical Systems*, April 2021

Instructions:

The 'take-home exam' consists of three parts:

- Part 1: math questions
- Part 2: essay questions about the course content
- Part 3: theoretical and practical questions alongside assignments in Matlab/Simulink, reminiscent to the assignments you already did before

Note that in terms of time investment and obtainable points these three parts are obviously not proportionally balanced.

Important for all parts: Do not solely present final answers but always provide argumentation. Where applicable also support your answers with calculations and/or equations (using an equation editor). Note that sometimes a word limit is given, so the aim is to write concisely (i.e., brief, though complete, and most of all clear and readable).

Construct a report with your answers, figures, discussions etc.. As always, adhere to the usual layout guidelines for academic manuscripts (e.g., clear font type with size > 10pts , 1.5 line distance, no columns, no figures to left or right side of main text, etc.). Hand in as pdf-file on Nestor at the designated submission portal. Also submit the ultimately used Simulink model file for the assignments, as well as the associated Matlab script with which you ran the model simulations. Please include your own name in the file names (e.g. HJdePoelExam.pdf.

Have fun and good luck!

Part 1: Math questions

1) Consider the following system for power production P: $\frac{dP}{dt} = -\eta P + \sigma P_{ext}$
in which P_{ext} indicates power supply from an external source. Determine the fixed point(s) and classify in terms of stability.

2) Determine the critical damping for the following system: $\frac{d^2\varepsilon}{dt^2} = -\zeta\frac{d\varepsilon}{dt} - \varepsilon/3$

3) In the course we considered the van der Pol oscillator. Another form of a non-linear oscillator is the so-called Rayleigh oscillator, which is rather similar to the van der Pol equation; the non-linear damping term is however velocity-dependent rather than position-dependent: $\ddot{x} + (\gamma + \epsilon\dot{x}^2)\dot{x} + \omega^2 x = 0$. To simplify, here we take $\epsilon = 1$. Now, examine the stability for this system.

Part 2: Essay questions

4) In the course we discussed several forms of dynamical system models and examined them in terms of stability. Given its general definition, provide a discussion of the concept of stability alongside the example of the Lorentz system. (max 200 words)

5) In the course we deliberated quite a bit on linear vs. non-linear systems. Discuss two important differences. (max 200 words)

6) In the final week you read the study of De Poel et al (2020). That article addresses a research question regarding the stability of rhythmic interlimb patterns. How did they quantify stability? Discuss the appropriateness of their measures alongside the adopted dynamical model. (max 250 words)

Part 3: Assignments

7) Coupled metronomes:
On Nestor (at 'files assignment 7') you can find the movement data from two metronomes coupled by a base, as digitized from a movie (see Nestor-> content). Analyze this data using relevant techniques that you learned during this course. Also perform simulations of a coupled oscillator system and analyze that data in the same way, so you can compare simulation outcomes with those of real data.
Now, your challenge is to find settings for your simulation model that yield approximately similar behavior as you see in the data of the real metronomes. Describe and discuss your findings. In doing so, support this by clear figures and a clear description of your strategy (i.e. the steps how you got to your eventual results). (max 250 words)

8) 'Epke dynamics':
Consider the giant circle on the high bar (or: horizontal bar) in gymnastics; as (sort of) announced in the lectures we are now going to model the dynamics of Epke Zonderland.

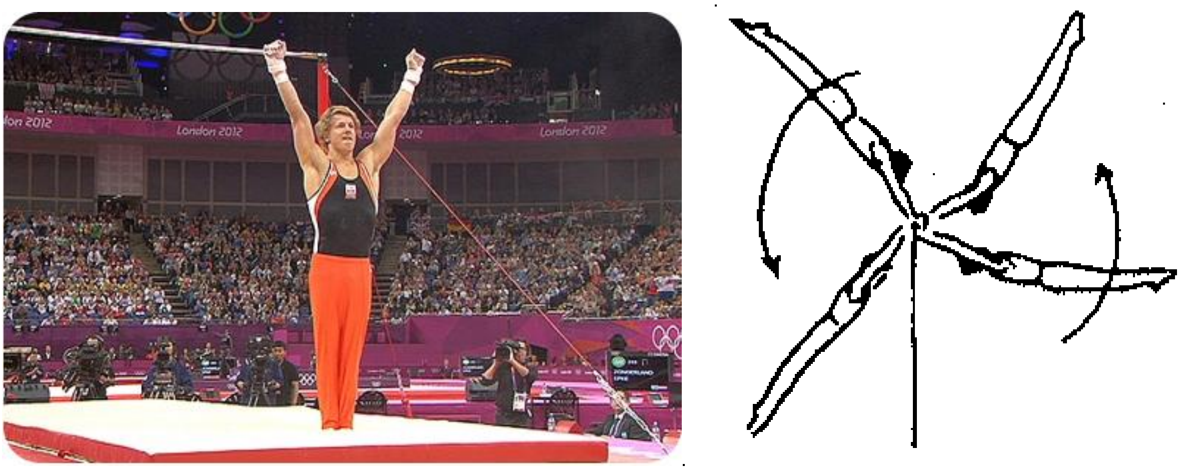

Figure 1: Photo of Epke Zonderland after finishing his gold medal performance in the London 2012 Olympic Games (left panel; you can almost hear reporter Hans van Zetten screaming ☺) and side view illustration of the giant circle move (right panel).

First, here's some essential information:

Epke's body measures are:

- Length: 173 cm
- Reaching height: 198 cm
- Weight: 69 kg

- Height of the horizontal bar above ground: 278 cm
(source: Wikipedia)

Now, adopt a Simulink model for simulating the behavior in the giant circle (see Figure 1, right panel). For this assignment you can assume that the mass of a human body is perfectly evenly distributed, and that the bar is rigid (i.e., it does not bend).

a) First, provide the particular differential equation (DE) that captures the movement around the bar for the specific case of Epke.

b) Consider the situation in which Epke's body is fully stretched during the whole cycle (see figure above, right panel); that is why it is called the 'giant circle'. We start this examination at an angle at which Epke is just about in handstand: Set the initial angle to 1 degree from the vertical. For the sake of simplicity of the assignment there is no damping. Given these settings, what is Epke's critical starting velocity to make at least one full giant circle? Determine this critical value *numerically* using simulations (note that your approximation should be sufficiently accurate). Report your strategy how you got to the answer and provide a clear figure of simulation results to support your answer. (max 200 words)

Figure 2: Photo of Epke Zonderland performing a stoop circle

c) Now, take the same settings as in b) but take a set value of the initial velocity that is just *slightly* higher than the critical value, so that our Epke-model makes at least one full giant circle. Now, after the first full giant circle, Epke decides to do a *stoop circle*: a technique which requires flexion of the body around the hip (see Figure 3: this is also known as 'Adler technique'). Effectively, this implies he temporarily changes his distance to the bar. How would this affect the dynamics of our Epke-system? First, discuss this theoretically alongside the things you learned during the course (max 200 words)

d) Now, apply the situation and settings in c) to your simulation and report a figure that shows the effects on the dynamics. In your figure, compare the outcomes for the stoop circle to those of the giant circle in a clear way. Briefly describe what can be seen from the figure. (max 100 words)